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**A KINETIC (NON-LINEAR) THEORY OF
TURBULENCE IN INCOMPRESSIBLE FLUIDS**

by

Toyoki Koga

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A KINETIC (NON-LINEAR) THEORY OF
TURBULENCE IN INCOMPRESSIBLE FLUIDS[†]

1. Two-Dimensional Case

by

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SUMMARY

In principle, a turbulence field is to be governed by the Navier-Stokes equations. In order to avoid the difficulty of treatment due to the non-linear characteristics of the Navier-Stokes equations, we begin with the assumption that a turbulence field may be represented by a proper distribution of many elementary vortex lines, each of which, being a particular solution of the Navier-Stokes equations, exhibits full characteristics of the non-linear equations. Based on this assumption, we introduce an equation which governs the distribution of those elementary vortex lines, in the same way as the Liouville equation governs the distribution of particles. With respect to a two-dimensional field, it is shown that Taylor's parabolic correlation mode for short distances and Kolmogoroff's $2/3$ -power correlation mode for moderate distances are unified to one correlation mode which is valid for the entire range of correlation distances. With respect to three-dimensional cases, introducing remarks are given in the appendix.

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Discussions with Dr. W. C. Thompson and Dr. R. L. Chuan were most helpful for materializing the theory.

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TABLE OF CONTENTS

<u>Section</u>		<u>Page</u>
I	Introduction	1
II	Elementary Vortex Lines	3
III	Kinetic Equations of Elementary Vortex Lines	5
IV	Modes of Interaction Among Elementary Vortex Lines	6
V	Correlation of Velocity Fluctuations Due to Clusters	9
VI	Discussions and Concluding Remarks	22
VII	References	25
	Appendix: Three-Dimensional Turbulence	27

1. INTRODUCTION

It has been widely felt that a satisfactory theory of turbulence must be one in which non-linear characteristics of fluid motion are fully taken into consideration. It is the purpose of this paper to propose a theory attempted from the above viewpoint. The medium is assumed to be incompressible, and detailed treatments are made only of a two-dimensional case.

As is well-known, the significance of velocity correlation as the cause of transport phenomena in a continuum was pointed out by O. Reynolds¹ in 1895, and clarified by G.I. Taylor² in 1921. As we see in works published successively by Taylor³, Prandtl⁴, von Karman⁵, Kolmogoroff⁶ and others between then and World War II, earlier treatments were mostly kinematical and phenomenological, being supported by similarity principle; physical characteristics of turbulence were described in terms of such quantities as mixing length, size of eddy, and so forth in those works. The nature of those characteristic lengths, as physical quantities, was known only vaguely and intuitively. Among those achievements, Taylor's experimental results of velocity correlation, his approach of coarse-graining the Navier-Stokes equations, Kolmogoroff's correlation functions have given strong influences on modern turbulence studies.

After the war, Heisenberg, Onsager, Burgers, Chandrasekhar and others⁷⁻⁸ advanced various theories in attempting to find feasible statistical laws governing the energy-spectrum characteristics of turbulence. The approaches of these authors were inductive; they conceived mutual interactions among vortices as the cause of thermalization or decay or irreversible transfer of energy among different spectrums, and attempted to formulate the transfer process by invoking statistical methods in the theory of Brownian motion.

In those treatments before and after the war, we see that those authors were assuming, if implicitly and intuitively, that turbulence is a manifest of vortexes of complex distributions.

Later, particularly in the last ten years, attempts have been made by many authors (for example, see Refs. 9 and 10) to derive feasible laws governing the irreversible process in a turbulence field by applying statistical treatments to the Navier-Stokes equations. Although the methods vary from one author to another, the principle is one: A turbulence field may be described precisely in terms of the velocity correlation factors of all the orders (of an infinite number). As was suggested by Taylor, von Karman and others previously, if one derives equations governing velocity correlation factors from the Navier-Stokes equations and solves them properly, one may achieve his goal. In this venture, however, one must meet great difficulties which are common in dealing with non-linear phenomena. Most usual methods of expansion of variables in series are not feasible, because of the difficulty of convergency.

We have had many experiences of overcoming the common difficulty of non-linear problems. As Eyring¹¹ wrote recently, science, in its various fields, has invented various models to make the best compromise between the infinite detail of reality and the limit of tractability. It should be remarked that, in general, a model of a system is made of integrals (invariants) of motion of the system. (Of course, approximate integrals of motion may be useful, too.) The present attempt of turbulence theory is made also in this sense.

The gists of the present attempt are 1) to take into consideration the effect of non-linear characteristics of dynamical processes governed by the Navier-Stokes equations, and at the same time 2) to avoid mathematical difficulties which are usual

in treating significantly non-linear fields.

In other words, this is an attempt to treat the structure of turbulence conceived by Heisenberg, Onsager and others in a deductive sense. The consideration of non-linear characteristics is believed to be essential in a deductive approach to turbulence theory, particularly when the turbulence is strong. While the proposed deductive approach to turbulence may appear to differ from those of other workers, it by no means contradicts or dismisses the existing understanding of turbulence as accumulated over the past fifty years. Rather, the present attempt is seen to constitute a natural and rational synthesis made of the knowledges achieved by the pioneers.

II. ELEMENTARY VORTEX LINES

There are two effects which characterize a flow field governed by the Navier-Stokes equations: 1) Non-linearity and 2) stress due to viscosity. We begin our investigation of turbulence by ignoring the effect of viscosity. This approximation is valid if the characteristic Reynolds number of turbulence is sufficiently large, insofar as turbulence is investigated in a free flow. (See section VI) On this approximation, we have to make an important choice between the following two views:

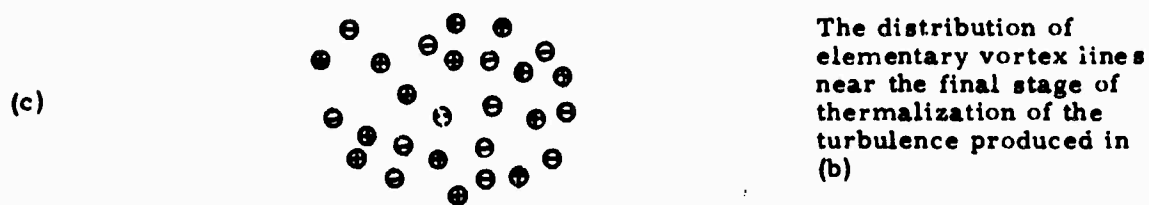
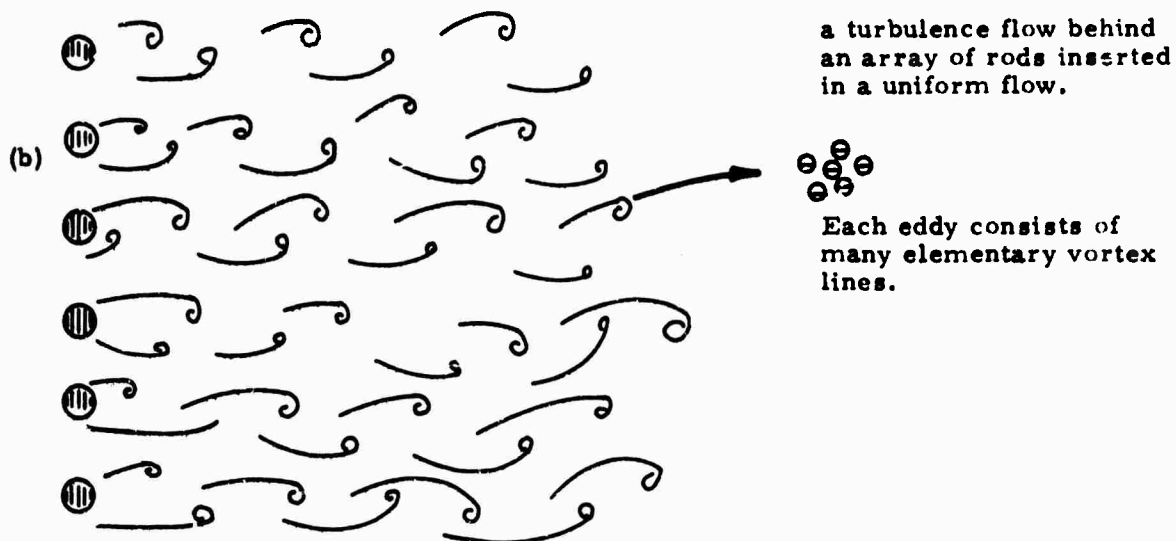
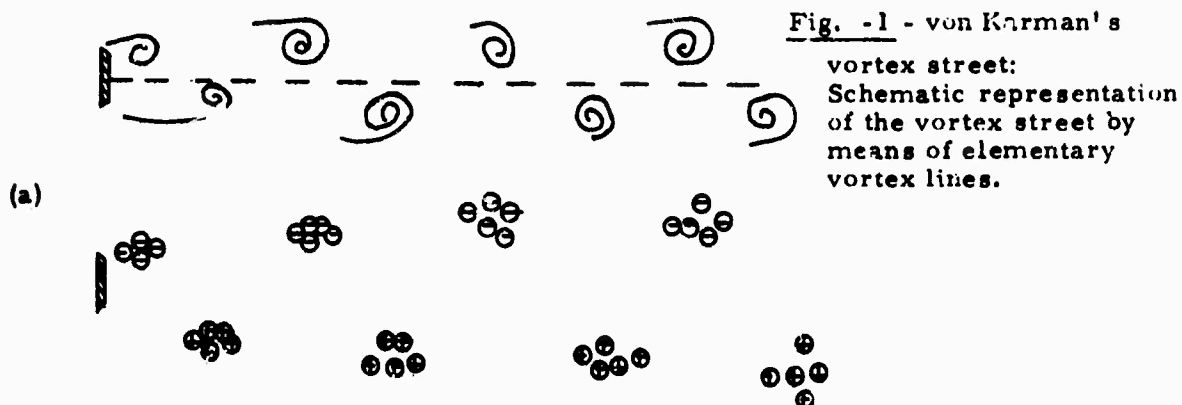
1. "A turbulence field is a field of vorticity, continuously but non-uniformly distributed in the configuration space. The strength of vorticity is finite. Therefore, there is no discrete vortex line, or tube conceived by Helmholtz. Based on this consideration, a turbulence field is to be treated as a continuous field of vorticity governed by the Navier-Stokes equation".

2. "Due to initial and boundary conditions, such as an array of rods inserted

in a flow which is otherwise uniform, however, aggregations of vorticity may exist. The field of vorticity may be continuous in the precise sense. But in an approximation, it is possible to represent the field as an ensemble of discrete vortex lines or tubes. By considering kinetic theory of such discrete vortexes, it may be possible to obtain essential characteristics of a turbulence flow, even if we may be ignoring the microscopic and detailed structure of the flow field."

As supporting the second approach, we have the following two knowledges: A vortex line, which may approximate a local aggregation of vorticity, is a particular solution of the non-linear Navier-Stokes equations, or the Euler equations on ignoring the viscosity effect, and is invariant as governed by Helmholtz's three laws. Secondly, we have a successful experience of kinetic theory of gases on the assumption that a gas consists of discrete and stable particles in spite of the fact that the most precise description of a gas as a whole is a quantum-mechanical wave function, which is spread in the entire space. Treating a gas as described by a wave function might appear precise, but in fact we may miss its essential characteristics due to its enormous complexity. Instead, the approximate model of particle structure enables us to see main characteristics of a gas rather easily. This is the approach of kinetic theory.

According to the above consideration, we assume that a turbulence field is represented by an appropriate distribution of elementary vortex lines; each elementary vortex line is of the same intensity and behaves according to Helmholtz's three laws. A turbulence field consists of such elementary vortex lines distributed non-uniformly. A vortex tube of a large size may be composed as a cluster of many elementary vortex lines of the same direction. It is noted that such a vortex tube of a large size exhibits a sort of thermalization phenomenon due to random migrations of elementary vortex lines constituting the tube, as is illustrated in Fig. 1.



III. KINETIC EQUATIONS OF ELEMENTARY VORTEX LINES

We assume that a turbulence field consists of N elementary vortex lines which are parallel to the z -axis and of which one half is of the positive sign and the other of the negative sign. We may define the distribution of the N vortex lines by

$$D^{(N)} = \pi \delta(\vec{r}_i - \vec{r}_i^*) \delta(\vec{\Omega}_i - \vec{\Omega}_i^*) \quad (3.1)$$

where \vec{r}_i^* is a function of time and denotes the position of vortex i , and $\vec{\Omega}_i^*$ denotes the direction and intensity of vortex i . Of course, $\vec{\Omega}_i^*$ is an invariant in the present case.

\vec{r}_i and $\vec{\Omega}_i$ are independent variables. If vortex j is at \vec{r}_j and induces velocity \vec{v}_{ij} at \vec{r}_i ,

$$\vec{v}_{ij} = \frac{(\vec{r}_i - \vec{r}_j) \times \vec{\Omega}_j}{(\vec{r}_i - \vec{r}_j)^2} \quad (3.2)$$

If \vec{V}_i is the velocity of the main (average) flow,

$$\vec{v}_i = \vec{V}_i + \sum_j \vec{v}_{ij} \quad (3.3)$$

is the total velocity at \vec{r}_i : the trajectory of vortex i at \vec{r}_i is governed by

$$\frac{d\vec{r}_i}{dk} = \vec{v}_i \quad (3.4)$$

Therefore $D^{(N)}$ defined by (3.1) satisfies

$$\left(\frac{\partial}{\partial t} + \sum_i \vec{v}_i \cdot \frac{\partial}{\partial \vec{r}_i} \right) D^{(N)} = 0 \quad (3.5)$$

The above equation is similar to the Liouville equation of N particles.

In a manner similar to that of the ordinary kinetic theory of particles, we may define

$$F^{(1)} = \int_{j \neq i} D^{(N)} d\vec{r}_i d\vec{\Omega}_i$$

$$F^{(2)}(i, j) = \int_{k \neq i, j} D^{(N)} d\vec{r}_k d\vec{\Omega}_k \quad (3.6)$$

and so forth. Consideration of

$$\frac{\partial}{\partial \vec{r}_i} \cdot \vec{v}_{ij} = \frac{\partial}{\partial \vec{r}_i} \cdot \frac{(\vec{r}_i - \vec{r}_j) \times \vec{\Omega}_j}{(\vec{r}_i - \vec{r}_j)^2}$$

leads to

$$\int \vec{v}_{ij} \cdot \frac{\partial}{\partial \vec{r}_j} D^{(N)} d\vec{r}_j \int \frac{\partial}{\partial \vec{r}_j} \cdot (\vec{v}_{ij} D^{(N)}) d\vec{r}_j = 0 \quad (3.7)$$

Hence, it is easily shown that Eq. (3.5) is reduced to

$$\left(\frac{\partial}{\partial t} + \vec{v}_i \cdot \frac{\partial}{\partial \vec{r}_i} \right) F^{(1)}(i) + \sum_j \vec{v}_{ij} \cdot \frac{\partial}{\partial \vec{r}_i} F^{(2)}(ij) dx_j = 0 \quad (3.8)$$

where

$$dx_j = d\vec{r}_i \cdot d\vec{r}_j \quad (3.9)$$

$$\begin{aligned} & \left[\frac{\partial}{\partial t} + (\vec{v}_i + \vec{v}_{ij}) \cdot \frac{\partial}{\partial \vec{r}_i} + (\vec{v}_j + \vec{v}_{ji}) \cdot \frac{\partial}{\partial \vec{r}_j} \right] F^{(2)}(ij) \\ & + \sum_k (\vec{v}_{ik} \cdot \frac{\partial}{\partial \vec{r}_i} + \vec{v}_{jk} \cdot \frac{\partial}{\partial \vec{r}_j}) F^{(3)}(i, j, k) dX_k = 0 \\ & \quad \quad \quad \left. \begin{matrix} i \\ j \end{matrix} \right\} = 1, 2, \dots, N \end{aligned} \quad (3.10)$$

and so forth. These equations constitute an indefinite series of equations in a manner similar to that of the Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy of equations governing the distributions of the molecules constituting a gas.

IV. MODES OF INTERACTION AMONG ELEMENTARY VORTEX LINES

1. Microscopic Interaction. If the interaction between elementary vortex lines i and j is strong, and they are fairly remote from the other vortices, the interaction is almost binary. In this case, Eq. (3.10) yields

$$\left(\frac{\partial}{\partial t} + \vec{v}_{ij} \cdot \frac{\partial}{\partial \vec{r}_i} + \vec{v}_{ji} \cdot \frac{\partial}{\partial \vec{r}_j} \right) F^{(2)}(i, j) = 0$$

$(\vec{v}_i = \vec{v}_j = 0 \text{ is assumed})$ (4.1)

According to the above equation, two vortices i and j move along trajectories such as illustrated in Fig. 2.

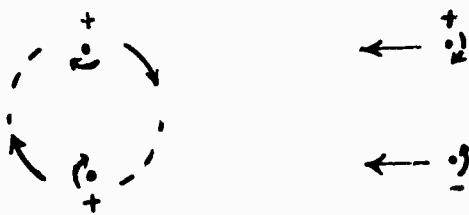


Fig. 2 - A pair of elementary vortex lines rotates if their signs are the same, but they move on two parallel lines if their signs are different.

If i and j are of the same sign, they move on a circle, while, if their signs are mutually opposite, they move on two parallel lines. Their interaction does not terminate unless they are disturbed by other elementary vortex line. We assume

that the strong correlation between i and j terminates in a finite and microscopic time period due to weak but frequent perturbations due to velocities exerted by other vortices such as represented by the last integral term in Eq. (3.11). According to (4.1), we obtain

$$\begin{aligned} \int \vec{v}_{ij} \cdot \frac{\partial}{\partial \vec{r}_i} F^{(2)}(i, j) d\vec{x}_j &= \int (\vec{v}_{ij} \cdot \frac{\partial}{\partial \vec{r}_i} + \vec{v}_{ji} \cdot \frac{\partial}{\partial \vec{r}_j}) F^{(2)}(i, j) d\vec{x}_j \\ &= \int \left(\frac{d}{dt} - \frac{\partial}{\partial t} \right) F^{(2)}(i, j) d\vec{x}_j \end{aligned} \quad (4.2)$$

where d/dt is an operator meaning the differentiation along the trajectories of the two vortices i and j . Since it is shown that

$$\frac{1}{\tau} \int_t^{t+\tau} \left(\int \frac{\partial}{\partial t} F^{(2)}(i, j) d\vec{x}_j \right) dt$$

is negligible, we obtain

$$\begin{aligned} \frac{1}{\tau} \int_t^{t+\tau} \left(\int \vec{v}_{ij} \cdot \frac{\partial}{\partial \vec{r}_i} F^{(2)}(i, j) d\vec{x}_j \right) dt \\ &= \frac{1}{\tau} \iint \frac{d}{dt} F^{(2)}(i, j) d\vec{x}_j dt \\ &= \int \left[\overline{F^{(2)}(t+\tau)} - \overline{F^{(2)}(t)} \right] d\vec{x}_j \\ &= \int \left[\overline{F^{(1)}(\vec{r}_i + \Delta \vec{r}_i) F^{(1)}(\vec{r}_j + \Delta \vec{r}_j)} \right. \\ &\quad \left. - \overline{F^{(1)}(\vec{r}_i) F^{(1)}(\vec{r}_j)} \right] d\vec{x}_j \end{aligned} \quad (4.3)$$

Here $\Delta \vec{r}_i$ and $\Delta \vec{r}_j$ are the distances traveled respectively by vortices i and j during a continuous correlation period of time τ . The approach deriving Eq. (4.3) and the interpretation are the same as those employed for the Boltzmann collision integral derived from the BBGKY hierarchy under the binary collision assumption, except that 1) the integrand in Eq. (4.3) is due to the non-uniformity of the vortex distribution in the configuration space while the integrand of the Boltzmann collision integral is due to the non-uniformity of the particle distribution in the momentum space, and that 2) the termination of the present interaction between vortex i and vortex j is

caused by disturbances of other vortexes, while the termination of a collision between two particles is due to the nature of Newton's dynamics and the relevant force law. (In case of charged particles, the termination of a binary collision is due to disturbances of other particles ¹².)

2. Interactions Among Semi-Stable Clusters of Elementary Vortexes.

A two-dimensional turbulence field is produced by an array of rods placed perpendicularly in a flow which is otherwise uniform. In origin, each rod produces a street of vortexes similar to von Karman's vortex street. Of course, there are interactions among those streets formed by many neighboring rods. Each vortex constituting the vortex streets is considered as a cluster of elementary vortex lines and may be initially simulated by a vortex tube of finite diameter instead of a vortex line. (see Fig. 1). Let us suppose that two vortex tubes are in interaction. Because of the finiteness of the diameter of a tube, their interaction is more complex than an interaction between two vortex lines.

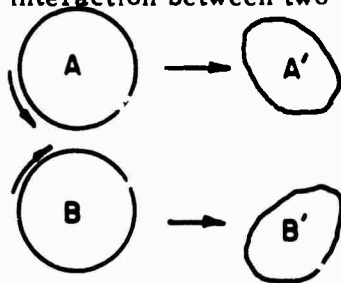


Fig. 3 - A is a cluster of elementary vortex lines of negative sign, and B is of positive sign. Initially, their cross-sections were circular. As they move on two parallel lines, their cross-sections are deformed: The part of A closer to B proceeds faster than the other part of A, etc.

In Fig. 3, two vortex tubes of different signs, A and B, are in interaction. They are mutually driven on two parallel lines in the first approximation, but the part of vortex A remote from B proceeds slower than the part close to B. Similarly, the part of B remote from A proceeds slower than the part close to A. Furthermore, each vortex tube is rotating by itself. As a result, the cross-section of each vortex tube is deformed as time passes. Similar and more complex deformations are caused on a vortex tube surrounded by many other vortex tubes. Thus a cluster (tube) decays from its outer fringe toward its center. As a cluster decays, those elementary vortexes which have drifted outward may be mixed with elementary vortexes of the

opposite sign which have drifted from another cluster. Those two groups of elementary vortexes of different signs may be mixed at random, and in effect the diameter of a vortex cluster becomes smaller. As this process of decay proceeds, each cluster becomes thinner and the effect of each cluster deforming other clusters becomes weaker. This situation is conceivable if we notice that a vortex line, instead of tube, is not deformed at all by other vortex lines. This may be the reason for the existence of an almost homogeneous and steady turbulence field indifferent of its initial method of formation. According to the above consideration, however, there is no homogeneous and steady turbulence in the strict sense. A seemingly homogeneous turbulence is still on the process of decay, although the rate of decay may be small.

3. Interactions of the Vlasov Type. If a distribution of elementary vortex lines is non-uniform in the microscopic sense, the integral term in Eq. (3.8) does not vanish even when the microscopic correlation between i and j is ignored; that is,

$$\sum_j \vec{v}_{ij} F^{(1)}(j) dx_j \cdot \frac{\partial}{\partial \vec{r}_i} F^{(1)}(i)$$

does not vanish and gives the effect of macroscopic interaction.

V. CORRELATION OF VELOCITY FLUCTUATIONS DUE TO CLUSTERS

As is described in the last section, clusters of elementary vortex lines are assumed to constitute a homogenous turbulence field. Based on this model, it would be possible to calculate the correlation between fluctuations at two different positions in the space. Such correlations were first observed experimentally by G. I. Taylor in 1935. A schematic correlation curve is given in Fig. 4.

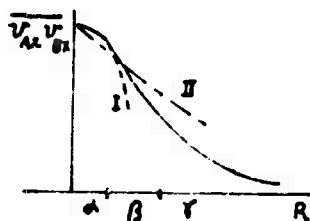


Fig. 4 - I; Parabolic law by Taylor; II; 2/3 power law by Kolmogoroff; the present theory gives the solid line.

Taylor could derive the correlation curve from the Navier-Stokes equations for distances between two positions which are short as compared with the size of eddy; that is the parabolic law:

$$\overline{v_{Ax} v_{Bx}} = \alpha(1 - R^2) \quad \text{I}$$

Later, Kolmogoroff gave a formula of correlation valid for a broader range of distances of two positions where the correlation is considered, by means of a dimensional consideration; that is

$$\overline{v_{Ax} v_{Bx}} = \alpha'(1 - R^{2/3}) \quad \text{II}$$

but so far, no rational attempt has been successful in deriving the correlation function which is valid over the entire domain of R . In the following, it will be shown that our present model is useful for the derivation.

We assume that each cluster consists of n elementary vortexes and is a circular cylinder of diameter d ; one-half of the number of the clusters is of the positive sign and the other half of the negative sign. They are distributed at random in the configuration space with a uniform density. We also assume that the measurement of velocities is made with respect to a coordinate axis system which is moving with the velocity of the main (average) flow. First, it will be shown that the velocity fluctuation at a position is attributed mainly to the nearest neighboring vortex cluster. Secondly, we calculate the velocity correlation at a pair of positions.

1. Localization of the Random Distribution of a Cluster. As discussed previously, a cluster of elementary vortex lines is produced by a proper boundary and/or initial conditions. Since the velocity of a cluster is finite and the time period in which a cluster passes through the field domain under investigation is also finite, the probability distribution of the cluster is localized. The local domains of space in each of which a cluster is present with uniform probability density are patched together and constitute the entire space domain under consideration. This consideration may be feasible, if the formation of the turbulence field is done in a proper way; as such

it is made with an array of rods. The linear dimension of such a local domain is denoted with l , which is of the order of the distance of two neighboring rods. By denoting the number density of clusters with n , $n^{-1/2}$ is of the order of the distance between two neighboring clusters. Assuming that an observer is moving with the velocity of the main flow V , and considering that the fluctuating part of velocity of a cluster v is smaller than V , we have

$$\frac{L \text{ (the distance which the observer travels during his observation)}}{V} = \frac{l}{v}$$

$$L \gg n^{-1/2}$$

Hence

$$l = \frac{Lv}{V} \gg \frac{v}{V} n^{-1/2}$$

Since $\frac{v}{V} < 1$, we may assume that

$$l \gtrsim n^{-1/2} \quad (5.1)$$

is the condition of our observation. This relation may be given based on von Karman's vortex street theory. This is also the condition that an arbitrary position in the field has the same probability, as the averaged one, to be occupied by a cluster. A similar condition is realized in the special case where each cluster is distributed with a uniform probability density all over the field. In both cases, the probability densities of clusters are the same. With respect to fluctuations in density, however, the two cases are different: In the former case, a space domain of linear dimension larger than l cannot be completely void of any one cluster at a moment of time. On the other hand, in the latter case, it is even possible that the entire field, except for one spot at least, is void of any cluster.

2. Significance of the Nearest Neighboring Cluster.

The most significant contribution of velocity fluctuation at a point p is made by the cluster which is the nearest neighbor of p . The reasons are:

1. The nearest distance and hence the largest induced velocity.

2. The location of a cluster is localized even though the precise location at each moment is unknown. We should notice the difference between the uniform probability distribution of a cluster and the localized probability distribution of the cluster. To explain the difference, we suppose that cluster a of strength w is localized in domain A and the cluster b is in domain B. If they are of the same sign and of the same strength, and A and B are mutually symmetric with respect to P where induced velocities are observed, the order of the fluctuation induced by the two clusters at P is

$$\frac{w}{2\pi L} \frac{\Delta \theta}{2} \quad (5.2)$$

where $\Delta \theta$ is the angle with which A and B is seen from P, and L is the approximate distance of A and/or B from P. On the other hand, if they are completely at random with respect to direction, the order of fluctuation is to be of the order of

$$\frac{w}{2\pi L} \quad (5.3)$$

In order to investigate the order of the magnitude of fluctuations at P induced by clusters surrounding the point, we assume that πl^2 is of the order of (density of clusters)⁻¹. Also, we assume that πl^2 is the area of the domain in which a cluster moves at random during the time period of our investigation.



Fig. 5 - The space is divided with co-centric circles of which the center is at P, where the velocity fluctuation is observed, and the radii are l , $3l$, $5l$, ...

We divide the space with co-centric circles of which the center is at P and the radii are l , $3l$, $5l$, ... The domain of the smallest circle is named domain 1. Domain 2 is the domain encircled by the smallest circle and its neighboring circle. Similarly we may define outwardly domain 3, domain 4, ..., domain 5, ... Domain 2, which is between the circle of radius l and radius $3l$ is divided by radial

lines into cells of the same area $\pi \ell^2$ which is the area of domain 1. The number of the cells is

$$\frac{\pi(3\ell)^2 - \pi\ell^2}{\pi\ell^2} = 8.$$

Suppose that a cluster of the same sign is confined in each cell. The velocity fluctuation at P induced by the cluster in domain 1 is of the order of

$$v_1 = \frac{w}{2\pi\ell/2}. \quad (5.4)$$

The total fluctuation of velocity at P induced by the eight clusters in domain 2 is of the order of

$$v_2 = \frac{w}{2\pi L_2} \Delta\theta_2 (2\Delta\theta_2) (4\Delta\theta_2)$$

where

$$L_2 = 2\ell, \quad \Delta\theta_2 = \frac{2\pi}{2 \times 8} = \frac{\pi}{8}.$$

It is easily shown that

$$v_2 = 0.11 \times v_1 \quad (5.5)$$

Similarly we may consider the effect of the clusters in domain 3. The number of the cells is

$$\frac{\pi(5\ell)^2 - \pi(3\ell)^2}{\pi\ell^2} = 16$$

and

$$v_3 = \frac{w}{2\pi L_3} \Delta\theta_3 (2\Delta\theta_3) (4\Delta\theta_3) (8\Delta\theta_3)$$

where

$$L_3 = 4\ell, \quad \Delta\theta_3 = \frac{2\pi}{2 \times 16} = \frac{\pi}{16}.$$

Hence

$$v_3 = v_2 \frac{1}{2} \left(\frac{1}{2}\right)^4 \times 8 = \frac{v_2}{4}. \quad (5.6)$$

In general, the effect of the clusters of the v th domain is given as follows: The number of the cells is

$$\frac{\pi(2^{v-1}\ell)^2 - \pi(2^{v-2}\ell)^2}{\pi\ell^2} = 8(v-1)$$

and

$$\Delta\theta_v = \frac{2\pi}{2 \times 8(v-1)} = \frac{\pi}{8(v-1)}, \quad L_v = 2^{v-1}\ell$$

Hence

$$v_v = \frac{w}{2\pi L_v} (\Delta\theta_v)^{k_v-1}, \quad k_v = 1, 2, \dots, 2^{v-1}. \quad (5.7)$$

Here

$$2^{k_v} = 8 (v - 1) \tag{5.8}$$

k_v as a function of v is tabulated as follows :

v	2	3	4	5	6	7	8	9	...
k_v	3	4	4<5	5	5<6	5<6'	5<6''	6	...

where $5<6$, for example, means a value between 5 and 6. It is an easy matter to see that

$$v_1, v_2, v_3, \dots$$

constitute a series which converges very quickly, and v_1 is a good approximation of the entire fluctuation.

3. Correlations of Velocities.

In order to see consequences of the above consideration, we calculate correlations between velocities at two positions A and B, induced by cluster C which is the common nearest neighboring one of the two positions A and B. It is assumed that A and B are moving with the main flow.

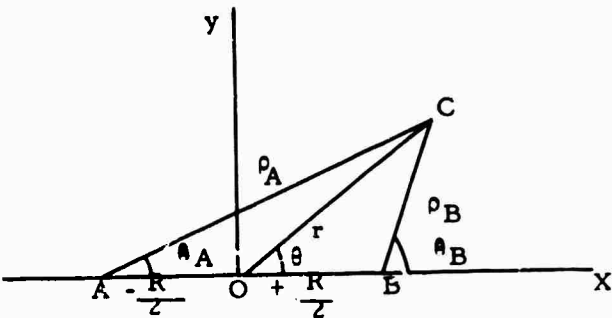


Fig. 6 - We calculate correlations between velocities at two positions A and B, induced by Cluster C

The distance between A and B is R . R is assumed to be changed from zero to infinity. If R is larger than ℓ , the nearest neighboring cluster of A is different from that of B. Then, in the approximation of ignoring those clusters other than the nearest neighboring one, there is no correlation between the fluctuations at A and B. We also assume that the radius σ of a cluster is finite and is sufficiently smaller than ℓ . A and B are on the x-axis with the origin of the coordinate system at the center of \overline{AB} . It is often convenient to give the position of C in terms of r , the

distance from the origin O and θ , the direction angle with respect to the x-axis.

Also we define

$$P_A = \overline{AC}, \quad P_B = \overline{BC}$$

$$\theta_A = \angle CAO, \quad \theta_B = \pi - \angle CBO.$$

1. The velocities at A and B.

(i) If $\rho_A \rho_B > 0$, we have for the velocities at A and B induced by cluster C of intensity ω

$$v_{Ax} = \frac{\omega}{2\pi\rho_A} \sin \theta_A$$

$$v_{Ay} = - \frac{\omega}{2\pi\rho_A} \cos \theta_A$$

$$v_B = - \frac{\omega}{2\pi\rho_B} \sin \theta_B$$

$$v_{By} = - \frac{\omega}{2\pi\rho_B} \cos \theta_B$$

It is a simple matter of geometry to find

$$\rho_A = \left[\left(\frac{R}{2} \right)^2 + r^2 + Rr \cos \theta \right]^{1/2}$$

$$\rho_B = \left[\left(\frac{R}{2} \right)^2 + r^2 - Rr \cos \theta \right]^{1/2}$$

$$\sin \theta_A = \frac{r \sin \theta}{\rho_A}, \quad \cos \theta_A = \frac{\frac{R}{2} + r \cos \theta}{\rho_A},$$

$$\sin \theta_B = \frac{r \sin \theta}{\rho_B}, \quad \cos \theta_B = \frac{-\frac{R}{2} + r \cos \theta}{\rho_B}$$

(5.9)

Hence

$$\begin{aligned}
 v_{Ax} &= \frac{\omega r \sin \theta}{2 \pi \left[\frac{R^2}{4} + r^2 + Rr \cos \theta \right]} \\
 v_{Ay} &= \frac{-\omega \left(\frac{R}{2} + r \cos \theta \right)}{2 \pi \left[\frac{R^2}{4} + r^2 + Rr \cos \theta \right]} \\
 v_{Bx} &= \frac{\omega r \sin \theta}{2 \pi \left[\frac{R^2}{4} + r^2 - Rr \cos \theta \right]} \\
 v_{By} &= \frac{-\omega \left(-\frac{R}{2} + r \cos \theta \right)}{2 \pi \left[\frac{R^2}{4} + r^2 - Rr \cos \theta \right]} \quad (5.10)
 \end{aligned}$$

(ii) If $\sigma > \overline{AC}$, we have

$$\begin{aligned}
 v_{Ax} &= \frac{\omega}{2 \pi \sigma} \times \frac{\rho_A}{\sigma} \sin \theta A \\
 &= \frac{\omega}{2 \pi \sigma^2} (r \sin \theta) \\
 v_{Ay} &= -\frac{\omega}{2 \pi \sigma^2} \left(\frac{R}{2} + r \cos \theta \right) \quad (5.11)
 \end{aligned}$$

Similarly, if $\sigma > \rho_B /$

$$\begin{aligned}
 v_{Bx} &= \frac{\omega}{2 \pi \sigma} r \sin \theta \\
 v_{By} &= -\frac{\omega}{2 \pi \sigma^2} \left(-\frac{R}{2} + r \cos \theta \right) \quad (5.12)
 \end{aligned}$$

In the following we calculate the average values

$$\overline{v_{Ax}^2}, \overline{v_{Ay}^2}, \overline{v_{Ax} v_{Ay}}, \overline{v_{Ax} v_{Bx}}, \overline{v_{Ax} v_{By}}, \text{ etc.}$$

when the cluster C changes its position around A and B during our observation.

It is readily seen, in view of the symmetry of the field, that

$$\overline{V_{Ax}^2} = \overline{V_{Ay}^2} = \overline{V_{Bx}^2} = \overline{V_{By}^2}$$

$$\overline{V_{Ax} V_{Ay}} = \overline{V_{Bx} V_{By}} = \overline{V_{Ax} V_{By}} = \overline{V_{Ay} V_{Bx}} = 0.$$

$$2. \overline{V_{Ax}^2}$$

$$\begin{aligned} \overline{V_{Ax}^2} &= \iint_I \left| \frac{w r \sin \theta}{2 \pi \left(\frac{R^2}{4} + r^2 + R r \cos \theta \right)} \right|^2 n r dr d\theta \\ &\quad + \iint_{II} \left| \frac{w r \sin \theta}{2 \pi \sigma} \right|^2 n r dr d\theta \\ &= (\overline{V_{Ax}^2})_I + (\overline{V_{Ax}^2})_{II} \end{aligned}$$

Here $n = \frac{1}{\pi l}$

and the domain of integration I is the domain between two co-centric circles of radius l and radius σ with center at A. Domain II is the domain encircled by the circle with radius σ and with center at A. Considering

(5.9), we have

$$\begin{aligned} (\overline{V_{Ax}^2})_I &= \iint_{\rho_A = \sigma}^{\rho_A = l} \left\{ \frac{w \sin \theta_A \rho_A}{2 \pi \rho_A^2} \right\}^2 n \rho_A d\rho_A d\theta_A \\ &= \frac{w^2}{4 \pi^2} n \int_0^{2\pi} \sin^2 \theta_A d\theta_A \int_{\sigma}^l \frac{1}{\rho_A} d\rho_A \\ &= \frac{w^2}{4 \pi} n \pi \log \frac{l}{\sigma} \end{aligned}$$

$$\begin{aligned}
 (\bar{v}_{Ax})_{II} &= \frac{\omega^2 n}{4\pi^2 \sigma^4} \int \sin^2 \theta_A d\theta_A \int_{\sigma}^{\rho_A} \rho_A^3 d\rho_A \\
 &= \frac{\omega^2}{4\pi^2 \sigma^4} n \pi \frac{\sigma^4}{4}
 \end{aligned}$$

Hence

$$\bar{v}_{Ax}^2 = \frac{\omega^2}{4\pi^2 \ell^2} \left[\log \frac{\ell}{\sigma} + \frac{1}{4} \right] \quad (5.13)$$

$$3. \overline{v_{Ax} v_{Ay}}$$

It is easily shown that this correlation factor vanishes:

$$\begin{aligned}
 \overline{v_{Ax} v_{Ay}} &= \int_{\rho_A=\sigma}^{\ell} \int_{\theta_A=0}^{2\pi} \frac{-\omega^2 \sin \theta_A \cos \theta_A}{4\pi^2 \rho_A} n d\rho_A d\theta_A \\
 &\quad + \int_{\rho_A=\sigma}^{\ell} \int_{\theta_A=0}^{2\pi} \frac{\omega^2 n}{4\pi^2 \sigma^4} \rho_A^3 \sin \theta_A d\theta_A d\rho_A \\
 &= 0.
 \end{aligned} \quad (5.14)$$

$$4. \overline{v_{Ax} v_{Bx}}$$

In order to calculate this factor, $v_{Ax} v_{Bx}$ is to be integrated over the domain of r , in which a point is within distances less than ℓ both from A and B. The manipulation may be complicated, depending on the relations among R , σ and ℓ .

(i). If $\ell < R$, A and B do not have any common nearest neighboring cluster. Hence

$$\overline{v_{Ax} v_{Bx}} = 0 \quad (5.15)$$

(ii). If $R > l$, $R - l > 0$, it is easy to calculate $\overline{v_{Ax} v_{Bx}}$. The integration is to be made over the domain \overline{abcd} indicated in Fig. 7. The maximum value of r for given θ is the solution of

$$r_m^2 + \left(\frac{R}{2}\right)^2 + 2\left(\frac{R}{2}\right) r_m \cos \theta = l^2$$

or

$$r_m = \frac{-R \cos \theta}{2} \pm l \left[1 + \frac{R^2}{8 l^2} (\cos^2 \theta - 1) \right] \quad (5.16)$$

We take only the positive value of r_m .

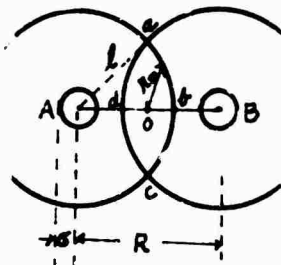


Fig. 7- If $R > l$, the clusters only in the domain \overline{abcd} contribute to the correlation between A and B.

Then we have

$$\overline{v_{Ax} v_{Bx}} = \frac{n \omega^2}{4 \pi^2} \int_{\theta=0}^{2\pi} \int_{r=0}^{r_m} \frac{r^2 \sin^2 \theta \, r \, dr \, d\theta}{\left[\frac{R^2}{4} + r^2 \right]^2 - R^2 r^2 \cos^2 \theta}$$

Since R is always larger than r , we expand the integrand in powers of r/R .

$$\overline{v_{Ax} v_{Bx}} = \frac{16 n \omega^2}{4 \pi^2 R^4} \iint r^3 \sin^2 \theta \left\{ 1 - 8 \frac{r^2}{R^2} + 16 \frac{r^4}{R^4} \cos^2 \theta + O \left[\frac{r^6}{R^6} \right] \right\} dr d\theta$$

Noting that

$$\int_0^{r_m} r^3 \, dr = \frac{1}{4} l^4 \left\{ 1 - \frac{2R}{l} \cos \theta + O \left[\frac{R^2}{l^2} \right] \right\}$$

$$\begin{aligned} \int_0^{r_m} r^5 \, dr &= \frac{1}{6} l^6 \left\{ 1 - \frac{3R}{l} \cos \theta + \dots \right\} \\ \iint r^3 \sin^2 \theta \, dr \, d\theta &= \frac{\pi}{4} l^4 \end{aligned}$$

$$\iint r \sin^2 \theta \left(-\frac{8}{R^2} + 16 \frac{\cos^2 \theta}{R^2} \right) dr d\theta$$

$$= -\frac{2}{3} \pi \frac{l^6}{R^2}$$

etc.

we obtain

$$\overline{v_{Ax} v_{Bx}} = \frac{16 n_0^2}{4 \pi^2 R^4} \left(\frac{\pi}{4} l^4 - \frac{2}{3} \pi \frac{l^6}{R^2} + \dots \right)$$

$$= \frac{\omega^2 l^2}{\pi^2 R^4} \left(1 - \frac{8}{3} \frac{l^2}{R^2} + \dots \right) \quad (5.17)$$

(iii) $R \ll \sigma < l$. The condition is illustrated in Fig. 8. A and B have a common nearest neighboring cluster. The domain where $r < \sigma$ is called domain I.

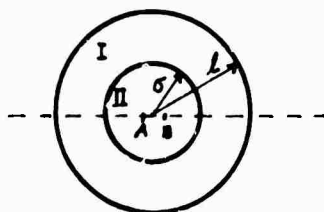


Fig. 8 - If $R \ll \sigma$, A and B have a common nearest neighboring cluster.

If C is in domain I, we have

$$\frac{r^2 \sin^2 \theta}{\left[\frac{R^2}{4} + r^2 \right]^2 - R^2 r^2 \cos^2 \theta} = \frac{\sin^2 \theta}{r^2} \left\{ 1 - 2 \frac{R^2}{4 r^2} + \frac{R^2}{r^2} \cos^2 \theta + O\left[\frac{R^4}{r^4}\right] \right\}$$

Hence,

$$\overline{(v_{Ax} v_{Bx})_I} = \frac{n_0^2}{4 \pi^2} \iint_I \frac{r^2 \sin^2 \theta d\theta dr}{\left[\frac{R^2}{4} + r^2 \right]^2 - R^2 r^2 \cos^2 \theta}$$

$$= \frac{\omega^2}{4 \pi^2 l^2} \left[\log \frac{l}{\sigma} + \frac{1}{8} R^2 \left(\frac{1}{l^2} - \frac{1}{\sigma^2} \right) \right]$$

If C is inside the domain II where $r < \sigma$, A and B are present within the core of a cluster. Hence,

$$(\overline{v_{Ax} v_{Bx}})_{II} = \frac{n \omega^2}{4 \pi^2 \sigma^2} \iint_{II} r^2 \sin^2 \alpha r dr d\theta = \frac{\omega^2}{16 \pi^2 \ell^2}$$

Therefore, we readily have

$$\overline{v_{Ax} v_{Bx}} = \frac{\omega^2}{4 \pi^2 \ell^2} \left[\log \frac{\ell}{\sigma} + \frac{1}{4} - \frac{R}{8} \left(\frac{1}{\sigma^2} - \frac{1}{\ell^2} \right) \right] \quad (5.18)$$

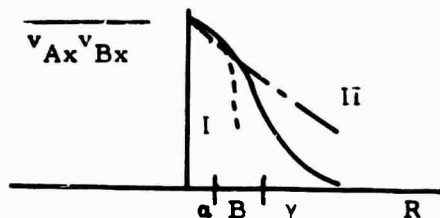


Fig. - 4 - I; Parabolic law by Taylor;
II; 2/3 power law by
Kolmogoroff; the present
theory gives the solid line.

$\overline{v_{Ax} v_{Bx}}$ given by (5.18) is valid for $R < \sigma$ and is in agreement with the experimental result obtained by Taylor (Fig. 4, α). $\overline{v_{Ax} v_{Bx}}$ given by (5.17) is valid for $R > \ell + \sigma$, and is in agreement with the part γ in Fig.

4. It is naturally expected that part β in the same figure must be obtained if we carry out the calculation, with no particular difficulty, by a means of a computing machine.

VI. DISCUSSIONS AND CONCLUDING REMARKS

1. The common difficulty in ordinary approaches of turbulence theory is that equations of correlation functions derived from the Navier-Stokes equations are not closed. The difficulty becomes serious due to the fact that the Navier-Stokes equations are non-linear. In order to have a closed set of equations or correlation functions, it is necessary to ignore some terms in some equations. In general, however, there is no assurance that those ignored terms are really ignorable without any significant errors.

2. By noticing that a vortex line is a singular solution of the Navier-Stokes equations where viscosity is ignored and that a vortex line is invariant in the sense of Helmholtz, we assume that a turbulence field constitutes many similar (elementary) vortex lines. On this assumption, we regard a turbulence field to be a discontinuous field of velocity. By so doing, however, the non-linearity of the field is tractable.

3. The effect of viscosity is ignored. One should note, however, that the effect of viscosity of a vortex line is not to extinguish any amount of vorticity; the vorticity concentrated on an elementary vortex line diffuses finally due to viscosity, but the total vorticity is invariant. If this broadening of a vortex line due to viscosity does not seriously affect the main characteristics of the field under consideration, it would be feasible to ignore the viscosity effect. The situation depends on the time and space scales on which a field is considered. It is well-known that the distribution of vorticity initially concentrated on a filament is given by

$$\omega = \frac{A}{t} \exp \left(-\frac{r^2}{4\nu t} \right)$$

where ν is (viscosity coefficient) / (fluid density), r the distance from the filament and t the time. (See L. Prandtl, the Mechanics of Viscous Fluids, in Aero-dynamic Theory edited by W. F. Durand, Vol. III, p. 68.) According to this formula, the order of the broadening of a filament is given by

$$r = (\nu t)^{1/2} \quad (6.1)$$

If r given here is sufficiently smaller than the linear dimension of our close observation l , our assumption of neglecting viscosity would be feasible. In this case, it would be proper to take for t the duration of our observation; that is,

$$t = \frac{l}{\bar{v}}, \quad l > r$$

where \bar{v} is the intensity of velocity fluctuation. Substitution of the above in (6.1) leads to

$$\frac{l \bar{v}}{\nu} > 1. \quad (6.2)$$

Our present treatment made by ignoring viscosity is feasible, if (6.2) is satisfied.

4. In a three-dimensional turbulence field, the behavior of a vortex line is complex and the equation of the distribution of vortex line is also complex as is shown in the Appendix.

5. The present article is merely a conceptual introduction. The treatment of the basic equation introduced here may vary significantly depending on the characteristics of boundary and initial conditions with which a turbulence field exists. The situation may be similar to that of the Boltzmann equation in cases of rarefied gas dynamics.

6. We note that there is a resemblance between the present theory and the ordinary kinetic theory of particles. Specifically, Eq. (3.5) is similar to the Liouville equation of many particle systems. In spite of the similarity of formalism, there is a significant difference between the present theory and the ordinary

kinetic theory of particles. At the kinetic stage of coarse-graining, before fluid-mechanical stage, the effect of interaction among vortices result in "diffusion" in the configuration space. On the other hand, the effect of interactions among molecules causes friction and diffusion in the momentum space, as is well seen in a Fokker-Planck type equation which may be derived from the Boltzmann equation.

7. It should be noted that a field of vorticity is not always a field of turbulence; it is possible for a field of vorticity to be stable. Phenomena in a stable field of vorticity, for example, laminar boundary layer, are out of the scope of the present investigation. Our investigation begins with the assumption that a field under consideration is turbulent.

VII. REFERENCES

1. O. Reynolds, Phil. Trans. Roy. Soc. 1895
2. G. I. Taylor, Diffusion by Continuous Movements, Proc. London Math. Soc. (2), 20, 196 (1921).
3. G. I. Taylor, Statistical Theory of Turbulence, Proc. Roy. Soc. London, Ser. A, 151, 421 (1935), 156, 307 (1936).
4. Th. von Karman and L. Howarth, On the Statistical Theory of Isotropic Turbulence, Proc. Roy. Soc. London Ser. A, 164, 192 (1938).
5. L. Prandtl, The Mechanics of Viscous Fluids, in Aerodynamic Theory vol. III edited by W. F. Durand, 1934.
6. A. N. Kolmogoroff, C. R. Acad. Sci. URSS, 30, 301 (1941).
7. W. Heisenberg, Zur Statistischen Theorie der Turbulenz, a. Physik, 124, 628 (1948).
8. C. C. Lin and W. H. Reid, Turbulent Flow, Theoretical Aspects, in Handbuch der Physik (Springer-Verlag, 1963), Band VIII/2.

9. R. H. Kraichnan, Phys. Rev. 99, 1407 (1958).
10. R. G. Deissler, Phys. Fluids 1, 111 (1958).
11. H. Eyring, Science, 150, 439 (1965).
12. T. Koga, Kinetic Equations for Plasmas, PIBAL Report No. 1007. 1967,
Polytechnic Institute of Brooklyn.
13. T. Koga, Some Development of Markoff's Method of Random Flights with the
Intention of Application to Turbulent Flow, PIBAL Report No. 954, 1965
Polytechnic Institute of Brooklyn.

APPENDIX THREE-DIMENSIONAL TURBULENCE

In a three-dimensional turbulence flow, an elementary vortex line is not a straight line; it forms often a closed line as is schematically illustrated in Fig.

9. Let us cut the closed line into segments of equal length ℓ .

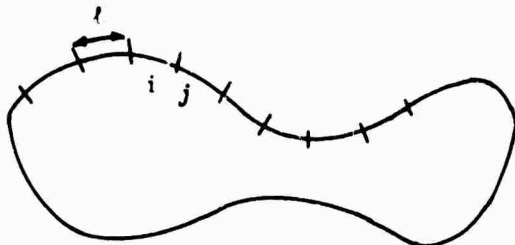


Fig. - 9 - A three-dimensional vortex line may be closed. We cut the closed line into segments of equal length, and investigate the distribution of those elementary segments.

We investigate the distribution of elementary segments. The strength of an elementary vortex segment is invariant, but the direction, the length ℓ and also the curvature change. If there is no large non-uniformity of velocity in the flow, ℓ may be almost invariant; if the segment is sufficiently short, we may ignore the curvature. By assuming these we have

$$\vec{v}_{ij} = \kappa \frac{(\vec{r}_i - \vec{r}_j) \times \vec{\Omega}_j}{(\vec{r}_i - \vec{r}_j)^2}$$

As similar to Eq. (3.2) here \vec{r}_i is the position of the center of elementary segment i and \vec{r}_j of elementary segment j ; κ is a function of ℓ which accounts for the finiteness of length ℓ . Then we have for the basic Liouville equation

$$\left[\frac{\partial}{\partial t} + \sum \vec{v}_i \cdot \frac{\partial}{\partial \vec{r}_i} + \sum (\Omega_i \cdot \frac{\partial \vec{v}_i}{\partial \vec{r}_i} \cdot \frac{\partial}{\partial \vec{\Omega}_i}) \right] D^{(N)} = 0 \quad (\text{A. 1})$$

as similar to Eq. (3.5).

The last term in the left hand side is the effect of changes in the directions of elementary segments; note that

$$\frac{d\vec{n}_i}{dt} = \frac{\Omega_i}{L_i} \vec{L}_i \cdot \frac{\partial \vec{v}_i}{\partial \vec{r}_i} = \vec{n}_i \cdot \frac{\partial \vec{v}_i}{\partial \vec{r}_i}$$

As is illustrated in Fig. 9, elementary segment i and its neighboring segment j which belong to the same vortex line have a correlation. This situation may be accounted for effectively by the consideration that the probable distribution of a segment is localized.

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13. ABSTRACT In principle, a turbulence field is to be governed by the Navier-Stokes equations. In order to avoid the difficulty of treatment due to the non-linear characteristics of the Navier-Stokes equations, we begin with the assumption that a turbulence field may be represented by a proper distribution of many elementary vortex lines, each of which, being a particular solution of the Navier-Stokes equations, exhibits full characteristics of the non-linear equations. Based on this assumption, we introduce an equation which governs the distribution of those elementary vortex lines, in the same way as the Liouville equation governs the distribution of particles. With respect to a two-dimensional field, it is shown that Taylor's parabolic correlation mode for short distances and Kolmogoroff's $2/3$ -power correlation mode for moderate distances are unified to one correlation mode which is valid for the entire range of correlation distances. With respect to three-dimensional cases, introducing remarks are given in the appendix.			

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